Measurement and Chemical Calculations.

Chapter 3

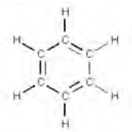
Measurement and Chemical Calculations

- Very large and very small numbers: exponential notation
- Metric system and SI base units
  - Mass, length, temperature, amount of material
  - Derived units (for other physical properties)
- Converting between units
- Calculations using dimensional analysis
- Quantifying and communicating uncertainty in measurements
- Significant figures and rounding

Very large and very small

- Numbers in chemistry tend to be much larger, and much smaller, than numbers in “every day life”
  - Number of atoms in 12.00g of carbon
    602214179000000000000000

  - Length of the bond between two carbon atoms in benzene
    0.000000000139 m



- We will express numbers using exponential notation
  - $6.02214179 \times 10^{23}$ (Avogadro's Constant)
  - 1.39×10^{-10} m

Exponential notation

B^p
B = base
p = power

$$10^4 = 10 \times 10 \times 10 \times 10 = 10000$$

$$10^{-4} = \frac{1}{10^4} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10000} = 0.0001$$

We generally use base 10, B=10
Large and small numbers are written as a.bcd x 10^e

one digit before the decimal point

Converting numbers into exponential notation: moving the decimal point

- Convert 724000 into standard exponential notation

We want $7.24 \times 10^{?}$

one digit before the decimal point

- How far do we need to move the decimal point?

724000.0 ⇨ 7.240000 moved 5 places left

Answer: 7.24×10^{5}

Reality check - 10^{5} is 100000 - a large number

Converting numbers into exponential notation: moving the decimal point

- Convert 0.000427 into standard exponential notation

We want $4.27 \times 10^{?}$

one digit before the decimal point

- How far do we need to move the decimal point?

0.000427 ⇨ 00004.27 moved 4 places right (different direction)

Answer: 4.27×10^{-4}

Reality check - 10^{-4} is 0.0001 - a small number

Exponential notation into "ordinary" decimal form

- Convert 4.71 x 10^{-4} into "ordinary" decimal form
- Note: it is a negative exponent so this is going to be a very small number (zeros after the decimal point)
- Move the decimal point 4 places to the left

00004.71 ⇒ 0.000471 moved 4 places

Exponential notation into "ordinary" decimal form

- Convert -7.2 x 10^{5} into "ordinary" decimal form
- Note: it is a positive exponent so this is going to be a very large number (zeros before the decimal point)
- Move the decimal point 5 places to the right

-7.200000 ⇒ -720000 moved 5 places

- Notice that the minus sign for -7.2 stays the same. -720,000 is a large negative number

The metric system

- Measurements everywhere in the world, with the exception of the US, Burma and Liberia are made in the metric system
  - the tiny island of St. Lucia converted to metric on April 1st 2008
- In the metric system units that are larger, or smaller, than the base unit are multiples of 10

g (grams), kg (kilograms = g x 1000) for weights

m (meters), km (kilometers = m x 1000) for distance

You can convert units by simply “moving the decimal point” and using exponential notation.

SI Units

- SI units are a subset of all metric units
- SI is an abbreviation for the French name for the International System of Units
- The SI system is defined by seven base units

Mass	kilogram
Length	meter
Temperature	kelvin
Time	second
Amount of substance	mole
Electrical current	ampere
Luminous intensity	candela


Antoine Lavoisier

Table 3.2 Metric Prefixes*

Large Units			*Small Units*		
Metric Prefix	**Metric Symbol**	**Multiple**	**Metric Prefix**	**Metric Symbol**	**Multiple**
tera-	T	10^{12}	Unit (gram, meter, liter)		$1 = 10^{0}$
giga-	G	10^{9}	deci-	d	$0.1 = 10^{-1}$
mega-	M	$1{,}000{,}000 = 10^{6}$	**centi-**	**c**	$\mathbf{0.01 = 10^{-2}}$
kilo-	**k**	$\mathbf{1{,}000 = 10^{3}}$	**milli-**	**m**	$\mathbf{0.001 = 10^{-3}}$
hecto-	h	$100 = 10^{2}$	micro-	μ	$0.000001 = 10^{-6}$
deca-	da	$10 = 10^{1}$	nano-	n	10^{-9}
Unit (gram, meter, liter)		$1 = 10^{0}$	pico-	p	10^{-12}

*The most important prefixes are printed in **boldface.**

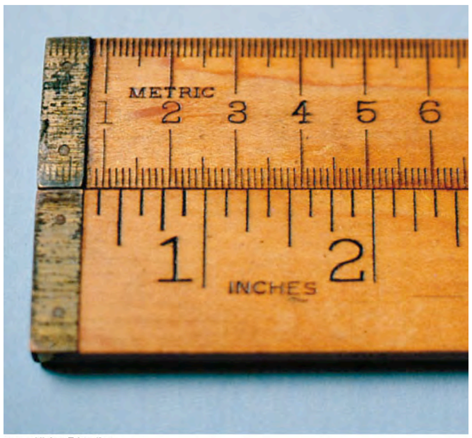
It is worth learning these (basic scientific literacy)

These are quantities that are very common in chemistry and biochemistry

Mass

- The SI unit of mass is the kilogram, kg
- It is defined as the mass of a platinum-iridium cylinder stored in a vault in France (!)
- It is the only SI unit that is still defined in relation to an artifact rather than to a fundamental physical property that can be reproduced in different laboratories
- A new 1kg sphere made of silicon is in the works. It will contain 2.15 x 10^{25} atoms

Length

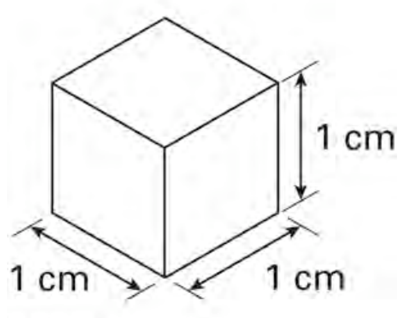


omson Higher Education

- The SI unit of length is the meter, m
- It is defined as the distance light travels in a vacuum in 1/299,792,468 second.

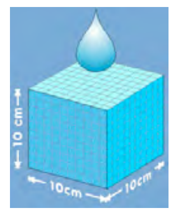
Volume

- The SI unit of volume is the cubic meter, m^3
- A more practical unit for laboratory work is the cubic centimeter, cm^3.

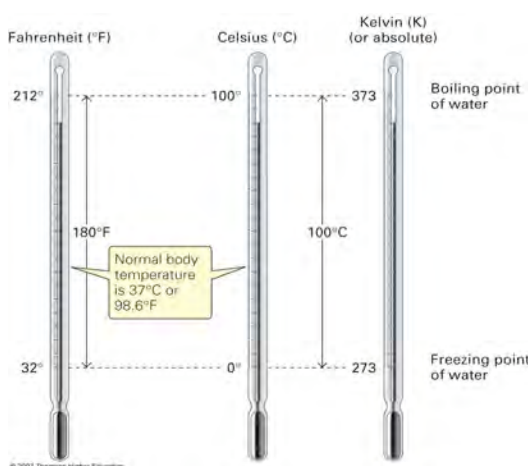


Volume

- One liter (L) is defined as exactly 1000 cubic centimeters
- Volume of a 10cm x 10cm x 10cm box
- One liter of water weighs 1000g = 1 kilogram
- 1 mL = 0.001 L = 1 cm^3



Temperature



- Fahrenheit Temperature Scale:
  - Water freezes at 32°F and boils at 212°F
- Celsius Temperature Scale:
  - Water freezes at 0°C and boils at 100°C
- T°F – 32 = 1.8 x T°C

Absolute temperature (kelvin)

- The other temperature scales have arbitary zero points
- Zero on the kelvin scale (0 K) is absolute zero - the lowest temperature possible
- Relationship between kinetic energy, the movement of particles, and temperature (in K)

$$E_K = \frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

- When the temperature is 0 K, all the particles/atoms in the material are stationary (v = 0)
- TK = T°C + 273

Amount of substance

- One mole is the amount of substance of a system which contains as many "elemental entities" (eg, atoms, molecules, ions, electrons) as there are atoms in 12 g of carbon-12: 6.022×10^{23}

1 mole carbon atoms

- You could use the mole as a unit of measurement for amounts of other things
  - eggs in SI units?
  - Not very practical!

1.9926×10^{-23} moles eggs
The 'dozen' is a better unit!

“Derived” units

- The SI units for all other physical property measurement are derived from their relationship to the 7 base units

- Examples
  - Common unit for force or weight is the Newton (N)
  - SI unit is $\frac{m \times kg}{s^2}$ or $m.kg.s^{-2}$
  - Common unit for pressure is the pascal (Pa), N/m^2
  - SI unit is $\frac{kg}{m \times s^2}$ or $m^{-1}.kg.s^{-2}$
  - Common unit for energy, heat or work is the joule, (J), Nm
  - SI unit is $\frac{m^2 \times kg}{s^2}$ or $m^2.kg.s^{-2}$

Example: units for density

- The mass and volume of a substance are directly proportional (more mass means more volume!),

$$m \propto V$$

- The proportionality is changed into an equation by inserting a proportionality constant, density

$$\text{mass} = D \times \text{volume} \quad \text{or} \quad D = \frac{\text{mass}}{\text{volume}}$$

- The mass per unit volume
- The SI units of density are kg/m^3, but we most often use g/cm^3 or g/mL
  - water has a density very close to 1 g/cm^3
  - osmium has a density of 22.6 g/cm^3

Example: units for density

- Notice that 1 $kg/m^3 \neq$ 1 g/cm^3

- 1 kg = 10^3 g
- 1 m^3 = 100 x 100 x 100 = 10^6 cm^3
- So

$$1\ kg/m^3 = \frac{10^3\ g}{10^6\ cm^3} = 10^{-3}\ g/cm^3$$

$$1 g/cm^3 = 1000\ kg/m^3$$

Density example calculations

- A piece of wood has a mass of 35 g. If its volume is 7 cm^3, What is its density?

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{35\ g}{7\ cm^3}$$

$$= 5\ g/cm^3$$

- Don't forget the units!

Density example calculations

Solving for mass

- The density of water is 1 g/mL. What mass does 30 ml of water have?

Mass = density x volume

$$= \frac{1\ g}{\cancel{mL}} \times 30\ \cancel{mL}$$

= 30 g

- Note that the mL canceled leaving only g, which is the correct unit for mass.

Density example calculations

Solving for volume

- The density of a substance is 7.8 g/cm^3. What volume does 39 g of it take up?

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{39\ g}{7.8\ g/cm^3}$$

= 5 cm^3

Dimensional analysis

- A quantitative problem-solving technique featuring algebraic cancellation of units and the use of PER expressions.
- PER: A mathematical statement of two quantities that are directly proportional to one another

How to solve a conversion problem by dimensional analysis

How many days are in 23 weeks?

- Identify and write down the given quantity. Include units.
  - Given: 23 weeks
- Identify and write down the units of the wanted quantity
  - Wanted: days
- Write down the PER/Path

  PER 7 days/week

  Path weeks ⟶ days

How to solve a problem by dimensional analysis

How many days are in 23 weeks?

- Write down the calculation setup

$$23\ \cancel{\text{weeks}} \times \frac{7\ \text{days}}{\cancel{\text{week}}} = 161\ \text{days}$$

- This gives you the correct answer and the correct units

PER 7 days/week
Path weeks → days

Dimensional analysis example

How many milliliters are in 0.00339 liter?

GIVEN: 0.00339 L

WANTED: mL

PER: 1000 mL/L

PATH: L → mL

$$0.00339\ \cancel{\text{L}} \times \frac{1000\ \text{mL}}{\cancel{\text{L}}} = 3.39\ \text{mL}$$

Reality check:
More mL (smaller unit) than L (larger unit). OK.

Dimensional analysis example

How many meters are in 2608 cm ?

GIVEN: 2608 cm

WANTED: m

PER: $\frac{1}{100}\ \frac{m}{cm}$

PATH: cm → m

$$2608\ \cancel{cm} \times \frac{1\ m}{100\ \cancel{cm}} = 26.08\ m$$

Reality check:
Less meters (larger unit) than centimeters (smaller unit). OK.

Using non-metric numbers

How many yards are in 2608 inches ?

GIVEN: 2608 inches

WANTED: yards

PER: $\frac{1}{36}\ \frac{yd}{in}$

PATH: in → yd

$$2608\ \cancel{in} \times \frac{1\ yd}{36\ \cancel{in}} = 72.444\ yd$$

In the metric calculation we just had to move the decimal point two places!

Dimensional analysis example

How many milliliters are in 1.0 quart?

GIVEN: 1.0 qt, *WANTED:* mL

PER: 1/1.06 L/qt, 1000 mL/L

PATH: qt → L → mL

$$1.0\ \cancel{qt} \times \frac{1\ \cancel{L}}{1.06\ \cancel{qt}} \times \frac{1000\ mL}{\cancel{L}} = 9.4 \times 10^{2}\ mL$$

Reality check:
More mL (smaller unit) than quarts (larger unit). OK
All units cancel leaving just mL. OK.

Units for Density

- If you wanted to publish your laboratory results, or to discuss them with non-chemists you might need to convert g/cm^3 into kg/m^3 (SI units)
  - There are 1000 g per kg (or 0.001 kg per g)
  - There are 1000000 cm^3 per m^3
- Define the path for the conversion

g/cm^3 —(0.001 kg/g)→ kg/cm^3 —(1000000 cm^3/m^3)→ kg/m^3

- Convert the density of lead, 11.4 g/cm3 into SI units

$$\frac{11.4\ \cancel{g}}{\cancel{cm^3}} \times \frac{0.001\ kg}{\cancel{g}} \times \frac{1000000\ \cancel{cm^3}}{L} = \frac{11400\ kg}{m^3}$$

Measurement and Chemical Calculations

- Very large and very small numbers: exponential notation
- Metric system and SI base units
  - Mass, length, temperature, amount of material
  - Derived units (for other physical properties)
- Converting between units
- Calculations using dimensional analysis
- Quantifying and communicating uncertainty in measurements
- Significant figures and rounding

Uncertainty in Measurement

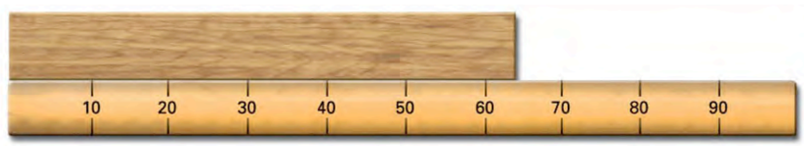
- No measurement is exact
- In scientific writing the uncertainty associated with a measured quantity is always included
- By convention, a measured quantity is expressed by stating all digits known accurately plus one **uncertainty digit**
- We describe the accuracy of a number by saying that it is known to “n” significant figures where “n” is the number of digits that are known accurately, plus one (the last digit, which was estimated)

The bottom board is one meter long.
How long is the top board?

More than half as long as the meter stick,
but less than one meter—about 6/10 of a meter.

The uncertain digit is the last digit written.

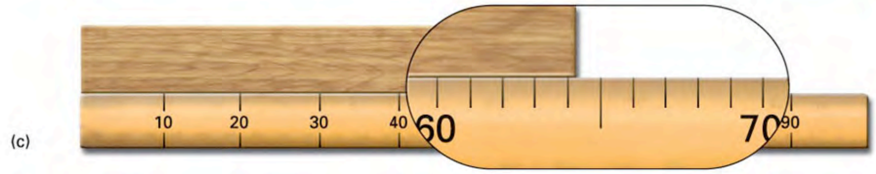
0.6 m



Now the meter stick has marks every 0.1 m,
numbered in centimeters. How long is the board?

Between 0.6 m and 0.7 m (with certainty),
and the uncertain digit must be estimated.

0.64 m

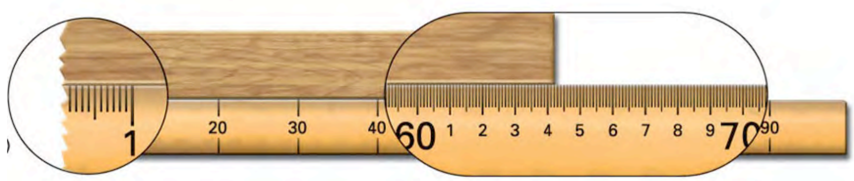


The measuring device now has centimeter marks.

How long is the board?

0.643 m

Estimating the last digit



The measuring device has millimeter marks.

We could estimate between the millimeter marks, but alignment of the board and the meter stick has an uncertainty of a millimeter or so.

We have reached the limit of this measuring device.

0.643 m
The measurement is accurate to three significant figures

Significant Figures

- The measurement process, not the units in which the result is expressed, determines the number of significant figures
  - The length of the board in the previous illustrations was 0.643 m. Expressed in centimeters, it is 64.3 cm
  - They are the same measurement with the same uncertainty. Both must have the same number of significant figures
- The location of the decimal point has nothing to do with significant figures
  - The same 0.643 m is 0.000643 km. The three zeros before the decimal point are not significant
  - Begin counting significant figures at the first nonzero digit, not at the decimal point

Examples

- How many significant figures?

23.5g	3 significant figures
10400 m	3 significant figures
0.03679 μl	4 significant figures
2.5×10^{-9} m	2 significant figures

Significant figures

- The uncertain digit is the last digit written
- If the uncertain digit is a zero to the right of the decimal point, that zero must be written
- If the mass of a sample is shown on the display of a balance as 15.10 g, and the balance is accurate to ±0.01 g, the last digit recorded must be zero to indicate the correct uncertainty

Rounding a calculated number

- If your calculator provides 10 digits, but the measurement or calculation method only gives an accuracy of 3 significant figures you will need to round your answer
- If the first digit to be dropped is less than 5, leave the digit before it unchanged

Examples
(round to 3 sig figs)

1.743345975 m → 1.74 m

0.041237856 kg → 0.0412 kg

Rounding a calculated number

- If your calculator provides 10 digits, but the measurement or calculation method only gives an accuracy of 3 significant figures you will need to round your answer
- If the first digit to be dropped is 5 or more, increase the digit before it by 1.

Examples
(round to 3 sig figs)

32.88 mL → 32.9 mL

0.009776 km → 0.00978 km

Using correct significant figures in calculations

The mass of 1.000 L of a gas is 1.436 g, what is the mass of 0.0573 L?

GIVEN: 0.0573 L *WANTED:* g

PER: $\frac{1.436\ g}{1.000\ L}$

PATH: L → g

$$0.0573\ \cancel{L} \times \frac{1.436\ g}{1.000\ \cancel{L}} = 0.0822828\ g$$

But: the volume 0.0573 is only known to 3 significant figures
The mass calculated using that volume cannot be more accurate
Answer: 0.0823 g

Using correct significant figures in calculations

How many hours are there in 6.924 days?

GIVEN: 6.924 days *WANTED:* hours

PER: 24 hours / day

PATH: days → hours

$$6.924 \text{ days} \times \frac{24 \text{ hours}}{1 \text{ day}} = 166.2 \text{ hours}$$

There are exactly 24 hours in one day. Exact numbers are infinitely significant. They never limit the # sig. figs.

Check: numbers of hours larger than number days, OK

Significant figure rule for multiplication and division

- Note which number in the calculation has the smallest number of significant figures
- Round off the answer to the same number of significant figures

- Example: calculate the volume of a box that is 34.49cm long, 23.0cm wide, and 15cm high

Volume = length x width x height

= 34.39 cm x 23.0 cm x 15 cm

4 s.f. 3 s.f. 2 s.f

Numerical answer = 11,899.05 cm^3 ← Note units

Round to 2 s.f. = 1.2×10^4 cm^3